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Comparison of nonlinear statistical growth models for describing rice (*Oryza sativa*) production in Gujarat

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Abstract
Rice is staple food of India. In Gujarat it is grown on large scale in South and Middle regions. Keeping in view the importance of rice crop, the present research deals with critical study of rice production of Gujarat using nonlinear statistical models and instability index analysis. Different nonlinear growth models *viz.* Monomolecular, Logistic, Gompertz and Richards model have been applied on rice production data considering the period 1980-81 to 2009-10. The validation of the best fitted models were carried out using data from 2010-11 to 2015-16. The parameters of these models were estimated using Levenberg-Marquardt's method. Model diagnostics of residuals of fitted models were carried out using run test and Shapiro-Wilk test. Validation of fitted models were carried out using different goodness of fit statistic like R^2, RMSE, MAE and MSE. The study found that Richards model described variation in production of rice crop. The research revealed that growth rate for production was found 5 per cent. The values of CDI for production of rice were 20.92 which revealed that rice production had medium instability in Gujarat.

Keywords: rice, richards model, shapiro-wilk test, run test

Introduction
Rice is a staple food of India as well as world. India ranks first in area while second in production after China for rice crop. The production of rice in India was 104.32 million tonnes for the year 2015-16 (Anonymous, 2016) [1]. In the Gujarat, rice was grown on an average about 763 thousand hectare with production of 1670 thousand tonnes and productivity of 2189 kg per hectare for the year 2015-16 (Anonymous, 2016) [1]. Most of the rice growing areas are confined to Middle and South Gujarat region.
Statistical modelling essentially consists of constructing models, represented by set of equations to describe the input-output relationship among the variables of interest. These statistical models helps in understanding trends of data and also facilitates determination of growth rate. Khan *et al.* utilized different nonlinear models to evaluate trends in area, production and productivity of rice in Chhattisgarh. In present investigation, statistical models were developed to study growth patterns of rice production in Gujarat.
Mere focus on growth gives the incomplete picture of growth scenario. Instability analysis needs to be carried out along with study of growth models. In this study, instability analysis has been carried out by using Cuddy-Della Valle index (CDI) for production of rice in Gujarat.

Material and Methods
In the present study the time series data of rice production were considered from the year 1980-2016 for the Gujarat which were obtained from website of Directorate of Economics and Statistics, Department of Agriculture, Cooperation and Farmers Welfare, Ministry of Agriculture and Farmers Welfare, Government of India. Different growth models were fitted on secondary data from the year 1980-81 to 2009-10 and the remaining data were used (*i.e.* from the year 2010-11 to 2015-16) for the validation of the fitted models.

A) Nonlinear growth model
In present investigation four different nonlinear growth models were taken into consideration. They were:
1) Monomolecular model (Draper and Smith, 1998) [3]

$$Y_t = \alpha - (\alpha - \beta) * Exp(-\lambda * t), \beta = Y_0 \ldots\ldots \quad (1)$$

2) Logistic model (Winsor, 1932) [13]

$$Y_t = \frac{\alpha}{[1 + \beta * Exp(-\lambda * t)]}, \beta = \frac{\alpha}{Y_0} - 1 \ldots \quad (2)$$

3) Gompertz model (Gompertz, 1825) [4]

$$Y_t = \alpha * Exp\left(-\beta * Exp(-\lambda * t)\right), \beta = ln\left(\frac{\alpha}{Y_0}\right) \ldots \quad (3)$$

4) Richards model (Richard, 1959) [9]

$$Y_t = \frac{\alpha}{[1 + \beta * Exp(-\lambda * t)]^{\left(\frac{1}{\delta}\right)}}, \beta = \frac{\alpha^\delta}{{Y_0}^\delta} - 1 \ldots \quad (4)$$

Where,
Y_t production of rice in Gujarat as dependent variable
t is time trend (in years) as independent variable
$\alpha, \beta, \lambda, \delta$ and Y_0 are the parameters
α is the carrying capacity
β is the function of Y_0
λ is the intrinsic annual growth rate
Y_0 is the production at time t =0
δ is the added parameter in the Richards model

In literature there are three prominent methods to fit any nonlinear regression model *viz.* Linearization (Taylor series) method, Steepest descent method and Levenberg-Marquardt's method (Draper and Smith, 1998) [3]. Among these three methods Levenberg-Marquardt's method is the most widely used method as it overcomes the short comings of other methods (Marquardt, 1963) [7]. Many sets of different initial values for the parameters were tried out to ensure global convergence.
Model diagnostics needs to be carried in order to check the assumptions of nonlinear regression. Randomness and normality of residuals are the two main assumptions of any nonlinear regression model. To check the randomness of residuals run test was performed (Siegel and Centellen, 1988) [11]. Shapiro-Wilk test was performed to check to normality of residuals (Shapiro-Wilk, 1965) [10]. Moreover normality of residuals were also checked using normal Q-Q plots.
In order to compare fitting of different nonlinear models different goodness of fit statistics were worked out. These goodness of fit statistics were R^2, RMSE, MSE and MAE. The formulas regarding different goodness of fit statistics are furnished below.

Coefficient of determination (R^2) (Kvalseth, 1985) [6]

$$R^2 = 1 - \frac{Residual\ SS}{Corrected\ SS} \quad \ldots (5)$$

Mean Squared Error (MSE) (Ratkowsky, 1990) [8]

$$MSE = \frac{\sum_i^n (y_i - \hat{y_i})^2}{(n-p)} \quad \ldots (6)$$

Mean Absolute Error (MAE)

$$MAE = \sum_i^n |y_i - \hat{y_i}| / n \quad \ldots (7)$$

Root Mean Squared Error (RMSE)

$$RMSE = \left[\frac{\sum_i^n (y_i - \hat{y}_i)^2}{n}\right]^{\frac{1}{2}} \quad \ldots (8)$$

Instability index analysis
The simple coefficient of variation often contains the trend component and thus overestimate the level of instability in time series data characterized by long term trends. To overcome this problem, the Cuddy-Della Valle index (CDI) were used (Cuddy and Della-Valle, 1978) [2]. The formula of CDI is expressed as,

$$CDI = (C.V.) \times \sqrt{1 - Adj. R^2} \quad \ldots (9)$$

Where,
C.V. is coefficient of variation in percentage
Adj. R^2 is coefficient of determination adjusted by degree of freedom
The range of Cuddy-Della Valle index (CDI) are as follows (Sihmar, 2014) [12]:
1. Low instability = between 0 to 15
2. Medium instability = greater than 15 and lower than 30
3. High instability = greater than 30

Results and Discussion
Different sets of initial values were tried out to assure convergence of the nonlinear models. All the four models considered in study converged during analysis. The parameter estimates of all the models are depicted in table 1. It was observed that Logistic model had the highest carrying capacity while Richards model had the lowest. Table 1 revealed that the intrinsic growth rate varied from 0.0008 to 0.05.

Table 1: Parameter estimates of various model fitted on rice production of Gujarat

Parameters	Monomolecular	Logistic	Gompertz	Richards
α	32.61 E+4	21.11 E+4	14.84 E+4	69.21 E+4
β	460.744	41.12 E+5	7.984	67.86 E+3
λ	0.0008	0.032	0.004	0.050
δ	-	-	-	1.544

Model diagnostics of the fitted models were carried out using residual analysis. The results of run test and Shapiro-Wilk test are represented in table 2. The p-values (mentioned in parenthesis) for residuals of all the fitted models for both the test were observed to be greater than 0.05. Therefore, the result of both the tests were found non-significant and null hypothesis *i.e.*, residuals were normally and randomly distributed were accepted. The normal Q-Q plots for all the four models revealed that residuals were normally distributed. The normal Q-Q plots of all the four models were represented in figures 1 to 4.

Table 2: Test for randomness and normality of residuals of fitted models for rice production in Gujarat

Test	Monomolecular	Logistic	Gompertz	Richards
Run test	-1.301^{NS} (0.193)	-0.929^{NS} (0.353)	-0.929^{NS} (0.353)	-0.929^{NS} (0.353)
Shapiro-Wilk test	0.961^{NS} (0.558)	0.903^{NS} (0.474)	0.962^{NS} (0.564)	0.903^{NS} (0.474)

Values in parenthesis are the p-values

Six years data (2010-11 to 2015-16) were utilized for validation of the fitted models. The details of validation of different models are given in table 3.

Table 3: Comparison of different nonlinear models using goodness of fit statistics

Goodness of fit statistics	Monomolecular	Logistic	Gompertz	Richards
R^2	0.578	0.606	0.604	0.606
RMSE	309.42	194.62	302.76	178.49
MAE	289.64	158.55	282.20	137.79
MSE	191486.38	75758.77	183324.92	95577.23

As observed in table 3, R^2 values varied from 0.578 to 0.606. Logistic and Richards model were found to have highest R^2 (0.606). Thus, both the models explained 60.6 per cent of the variability present in rice production of Gujarat. It was found that RMSE, MAE and MSE ranged from 178.49 to 309.42, 137.79 to 289.64 and 75758.77 to 191486.38, respectively. The table revealed that Richards model had lowest value for MAE (137.79) and RMSE (178.49). Logistic model was found to have least value of MSE (75758.77).

On the basis of model diagnostics and validation of fitted models, Richards model best described the variability for rice production of Gujarat. The estimated values corresponding to observed value were computed by using Richards model are represented graphically in Fig. 5. From the figure it was observed that production of rice for the years 1985-86, 1986-87, 1987-88, 2000-01 and 2002-03 had a sudden decline due to drought (Gupta, 2011). The validation of the best fitted model *i.e.*, Richards model is shown in Fig. 6. As shown in table 1, Richards model had an intrinsic growth rate of 0.05 which depicts that the production of rice in Gujarat for the time frame of investigation had increased at an annual growth rate of 5 per cent.

Instability index was calculated for the time series data of production for 36 years from 1980-81 to 2015-16. Different statistics calculated related for instability analysis are represented in Table 4. The linear model was fitted on data of rice production against time trend and it was observed that the value of R^2 was 73.8 per cent. The coefficient of variation was observed to be 40.87 per cent. The CDI value for production was observed to be 20.92 which gives clear signs of medium instability.

Table 4: Instability index analysis

Particulars	Mean	S.D.	Adj. R^2	C.V.	CDI
Production	1012.1194	413.6912	0.738	40.87	20.9216

Conclusion

The research revealed that the growth rate of rice has was 5 per cent per annum with medium instability. Increase in area of cultivation, increase in irrigation area, introduction of high yielding varities, better management practices, increase in funding to agriculture and better linkage between research institutes and framers are th major reasons behind increase in production of rice in Gujarat

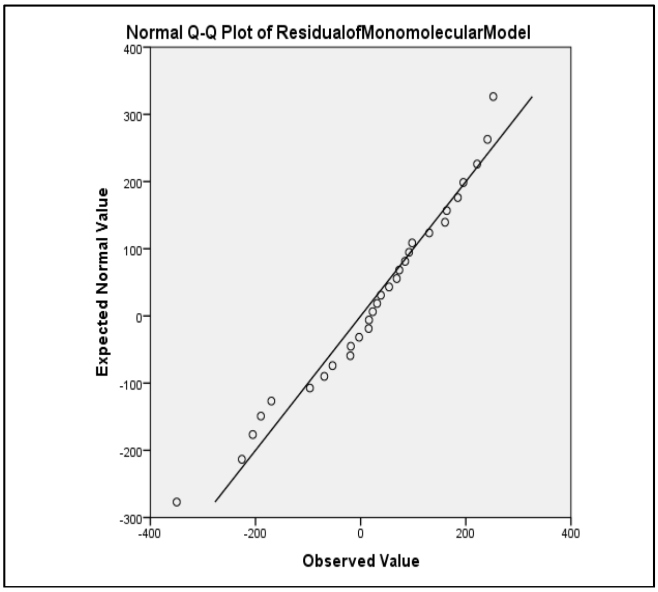

Fig 1: Normal Q-Q plot of residuals of Monomolecular model fitted on rice production of Gujarat

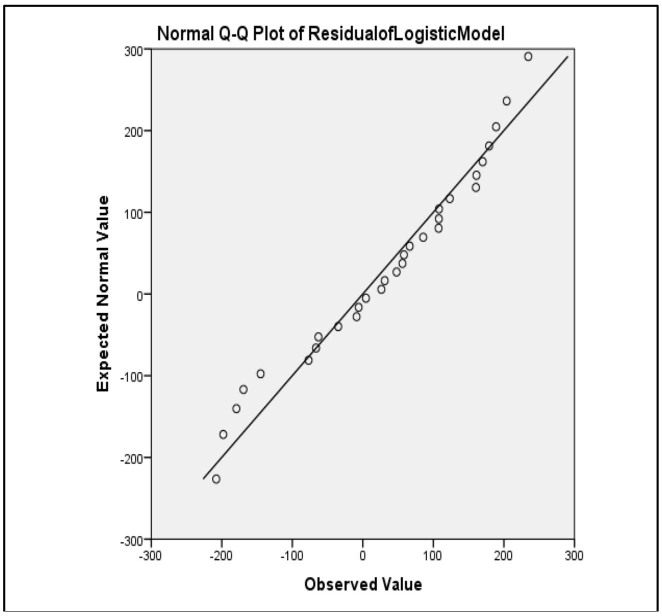

Fig 2: Normal Q-Q plot of residuals of Logistic model fitted on rice production of Gujarat

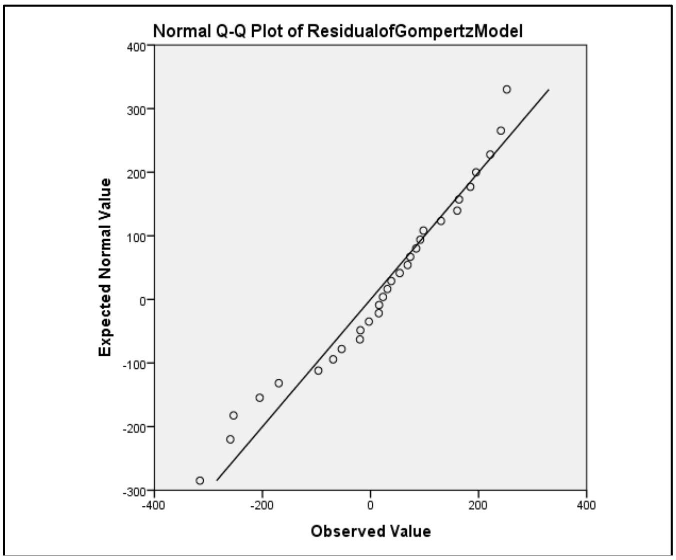

Fig 3: Normal Q-Q plot of residuals of Gompertz model fitted on rice production of Gujarat

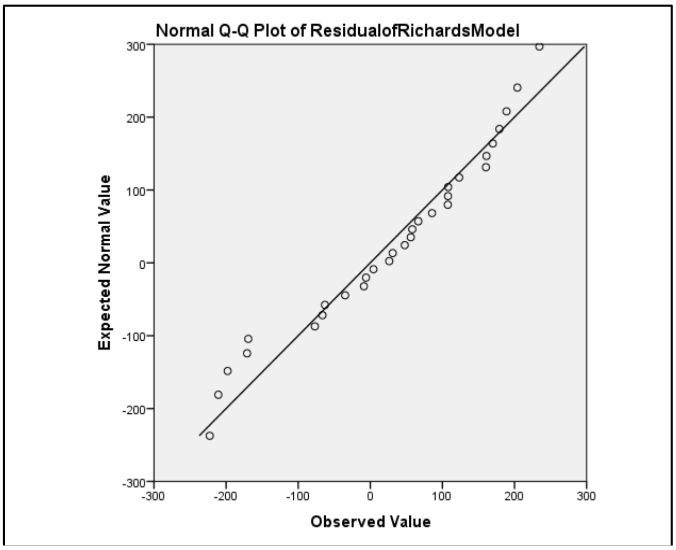

Fig 4: Normal Q-Q plot of residuals of Richards model fitted on rice production of Gujarat

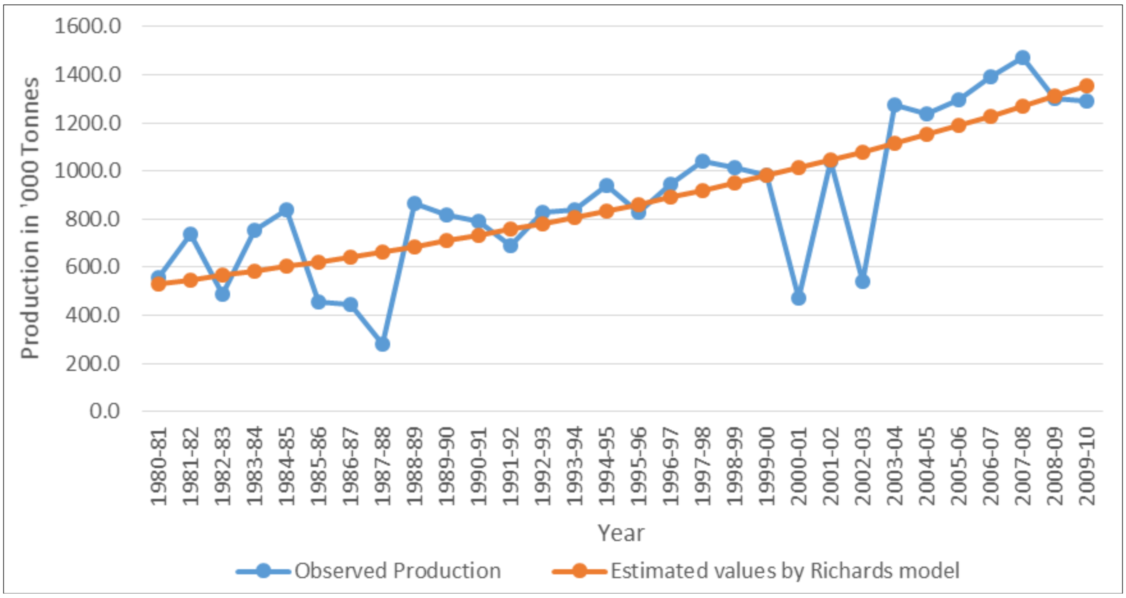

Fig 5: Graph of observed and estimated values of rice production for Gujarat using Richards model

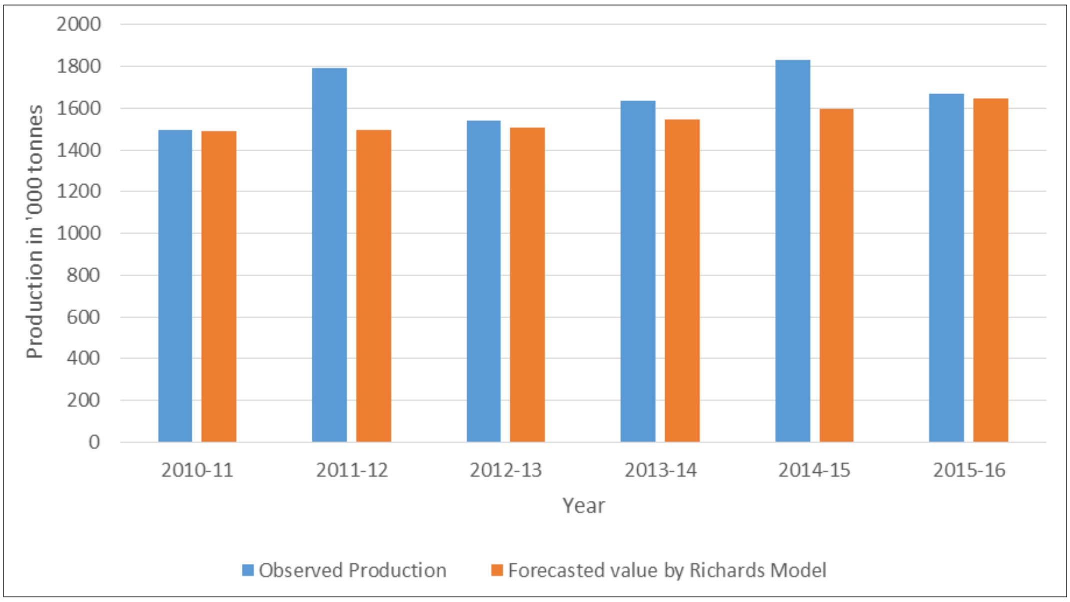

Fig 6: Observed and forecasted values of rice production for Gujarat using Richards model

References

1. Anonymous. Pocket Book of Agricultural Statistics. Government of India, Ministry of Agriculture & Farmers Welfare, Department of Agriculture, Cooperation & Farmers Welfare, Directorate of Economics & Statistics, New Delhi, 2016, 38-73.
2. Cuddy JDA, Della-Valle PA. Measuring the instability of time series data. Oxford Bulletin of Economics and Statistics. 1978; 40(1):79-85.
3. Draper NR, Smith H. Applied Regression Analysis. Edn 3, John Wiley, New York, 1998, 148-210.
4. Gompertz B. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. Philosophical Transactions of the Royal Society of London. 1825; 115(7):513-583.
5. Gupta RK. The role of water technology in development: a case study of Gujarat State, India. UN-Water International Conference, Spain, 2011, 1-2.
6. Kvalseth TO. Cautionary not on R^2. American Statistician. 1985; 39(11):279-85.
7. Marquardt DW. An algorithm for least squares estimation of nonlinear parameters. J. Soc. Ind. Appl. Math. 1963; 11(09):431-41.
8. Ratkowsky DA. Handbook of Nonlinear Regression Models, Marcel Dekker, New York, 1990, 150-215.
9. Richards FJ. A flexible growth function for empirical use. Journal of Experimental Botany. 1959; 10(61):290-300.
10. Shapiro SS, Wilk MB. An analysis of variance test for normality (complete samples). Biometrika. 1965; 52(34):591-611.
11. Siegel S, Centellen NJ. Nonparametric Statistics for Behavioural Science, Mcgraw-Hill. 1988.
12. Sihmar R. Growth and Instability in Agricultural Production in Haryana: A District level Analysis. International Journal of Scientific and Research Publications. 2014; 4(7):1-12.
13. Winsor CP. The Gompertz curve as a growth curve. Proceedings of the National Academy of Sciences of the United States of America. 1932; 18(19):1-8.